

Solutions

Exam 1 Chapters A-D and 1

Answer the following questions. You must show your work to receive full credit. Be sure to make reasonable simplifications. Give exact answers. Indicate your final answer with a box.

True or False (3 points each)

F 1. $\sqrt{9+4} = \sqrt{9} + \sqrt{4}$

$\sqrt{13} \neq 5$

T 2. $(a \cdot b)^3 = a^3 \cdot b^3$

Exponent Rules

F 3. $(8+7)^2 = 8^2 + 7^2$

$225 \neq 113$. Never Do This!!

T 4. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Multiplication of Fractions.

F 5. $\frac{x+4}{x-4} + \frac{x+5}{x+4} = \frac{(x+4)+(x+5)}{(x-4)+(x+4)}$

Try $x=1$.

T 6. $(x^{12})^4 = (x^4)^{12}$

Exponent Rules

T 7. If a and b are real numbers such that $a < b$ then a is to the left of b on the number line.

Ordering of \mathbb{R} .

Simplify Using Exponent Rules (6 points each)

1. $(u^6 v^3)^{1/3}$

"

$u^2 v$

2. $\sqrt{x \sqrt{x^3}}$

"

$$(x(x^3)^{1/2})^{1/2} = (x \cdot x^{3/2})^{1/2} = (x^{5/2})^{1/2} = x^{5/4}$$

Factor Completely The Expression(6 points)

3. $y(y + 9) - 6(y + 9)$

"
"
$$(y-6)(y+9)$$

Simplify The Rational Expression(6 points each)

4. $\frac{1}{x} + \frac{2}{x^2} + \frac{1}{x^3}$

"
"
$$\frac{x^2}{x \cdot x^2} + \frac{2 \cdot x}{x^2 \cdot x} + \frac{1}{x^3} = \frac{x^2 + 2x + 1}{x^3} = \frac{(x+1)^2}{x^3}$$

5. $\frac{y-3}{y^2-1} \cdot \frac{y^2+11y+10}{y^2+7y-30}$

"
"
$$\frac{y-3}{(y-1)(y+1)} \cdot \frac{(y+10)(y+1)}{(y+10)(y-3)} = \frac{1}{y-1}$$

Solve The Following Equations And Inequalities(6 points each)

6. $w^2 = 3(w - 1)$

$$w^2 - 3w + 3 = 0$$

$$\frac{3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{3 \pm \sqrt{-3}}{2} \quad (\text{No Real Solutions})$$

7. $(x + 2)^4 = 81$

$$\left((x+2)^4 \right)^{1/4} = (81)^{1/4}$$

$$x+2 = \pm 3$$

$$x = -2 \pm 3 = -5, 1.$$

8. A university music department plans to stage an opera. The fixed cost for the set, costumes, and lighting is \$ 5000, and they plan to charge \$ 15 a ticket. So if they sell x tickets, then the profit P they will make from the performance is given by the equation

$$P(x) = 15x - 5000$$

- (a) Find the net change when the number of tickets sold increases from 100 to 200. (3 points)
(b) Evaluate $P(1000)$ and explain what it means. (3 points)

$$\begin{aligned} (a) \quad P(200) - P(100) &= (15(200) - 5000) - (15(100) - 5000) \\ &= -2000 - (-3500) = \$1,500. \end{aligned}$$

$$(b) \quad P(1000) = 15(1000) - 5000 = \$10,000.$$

When the music department sells 1000 tickets they will make a profit of \$10,000.

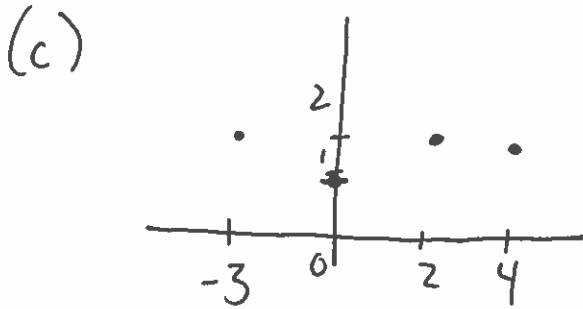
9. A set of ordered pairs defining a relation is given below.(9 points)

$$\{(-3, 2), (4, 2), (2, 2), (0, 1)\}$$

- (a) Find the domain of the relation.
- (b) Find the range of the relation.
- (c) Sketch a diagram of the relation.
- (d) Does the relation define a function?

(a) Domain = $\{-3, 4, 2, 0\}$

(b) Range = $\{2, 1\}$



(d) Yes, it passes the vertical line test.

10. Two entrepreneurs, Dirk Funk and Tom Gugliotta, drove 4500 miles from England to Timbuktu, Mali in a truck powered by chocolate. They used an ethanol that is made from old, unusable chocolate, and it took 17 pounds of chocolate to make 1 gallon of ethanol. The table below gives the data for the relationship between the amount of chocolate used and the number of miles driven. (9 points)

Miles Driven (mi)	Pounds of Chocolate Used (lbs)
0	0
20	17
40	34
60	51
80	68
100	85

- (a) Is a linear model appropriate? Justify your answer.
- (b) If so, determine the model for the relationship between pounds of chocolate used (as the output) and the number of miles driven (as the input).
- (c) How many pounds of chocolate are needed to drive 90 miles?
- (d) If Dirk and Tom only have 115 pounds of chocolate, how far can they drive?

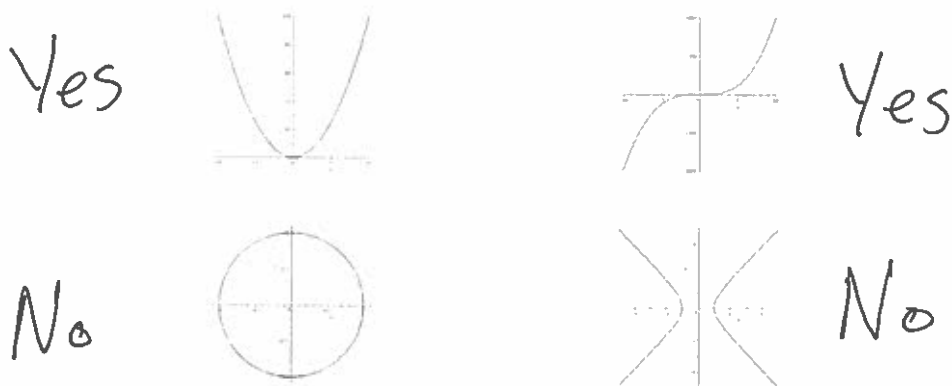
(a) Yes, everytime miles increases by 20, ~~the~~ lbs increases by 17; i.e. both have constant change.

(b) $y = \frac{17}{20}x$ where $x = \text{miles}$ and $y = \text{lbs}$.

(c) When $x = 90$, $y = \frac{17}{20} \cdot 90 = \frac{153}{2} = 76.5$ lbs of chocolate.

(d) When $y = 115$, $115 = \frac{17}{20} \cdot x \Leftrightarrow 115 \cdot \frac{20}{17} = x = 135.29$ miles.

11. Several graphs are given below. Determine which graphs are the graphs of functions. Justify your answers. (8 points)



All by Vertical Line test.

12. (5 points) Consider the function given by

$$g(w) = \frac{3(w-1)^2}{w-2}$$

- (a) What is the name of the function?
- (b) What letter represents the input?
- (c) What is the output?
- (d) Find $g(3)$. What does it represent?
- (e) What is the domain of the function?

(a) g

(b) w

(c) $g(w)$ or $\frac{3(w-1)^2}{w-2}$

(d) $g(3) = 12$, when the input is 3 the output is 12.

(e) Can't divide by zero

So $\text{Dom}(g) = \mathbb{R} - 2$

$= (-\infty, 2) \cup (2, \infty)$.